

The Crystal Structure of the *M* Phase, Nb–Ni–Al*

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The crystal structure of the *M* phase (Nb₄₈Ni₃₉Al₁₃) has been determined by single-crystal X-ray analysis. The lattice constants are $a_0 = 9.303(6)$, $b_0 = 16.266(10)$, $c_0 = 4.933(2)$ Å, and the space group is *Pnam* (D_{2h}^{16}). The crystal structure resembles that of the (Mo–Ni–Cr) *P* phase [space group *Pbnm* (D_{2h}^{16})], except that there are fewer atoms per unit cell (52 for the *M* phase, 56 for the *P* phase), the glide plane types are interchanged between the *a* and *b* axes, and five-membered rings replace all six-membered rings in the main layers. The structure has been refined by full-matrix least-squares analysis to a final *R* of 0.115, based on the 420 observed reflections. The atoms have characteristic coordination polyhedra with 12, 14, 15 and 16 vertices in the same relative numbers as in the μ phase. All positions with C.N.14, C.N.15, and C.N.16, are occupied essentially completely by Nb; the positions with C.N.12 are occupied by a mixture of Ni and Al and possibly some excess Nb. The atomic radii calculated from the observed distances are about 4% larger than those in the (Mo–Ni–Cr) *P* phase, the (Mo–Ni) δ phase, the (Mo–Co) μ phase, and about 5% smaller than those in the (Nb₂Al) σ phase. This structure turns out to be one of several predicted on geometrical grounds by Frank & Kasper (*Acta Cryst.* 12, 483).

Introduction

The *M* phase was discovered by Benjamin, Giessen & Grant (1966) in ternary systems of Nb and Ni with Al, Ga or Cu. The *M*-phase field in the Nb–Ni–Al system appears in the vicinity of a μ -phase field and both fields are elongated approximately parallel to the Ni–Al binary line, suggesting that Al and Ni occupy the same kinds of site in the structure. A Laves-phase (*C14*) field in the same diagram is elongated in the same direction. The powder diffraction diagram of the *M* phase is very complex and shows the same bunching of strong lines as has been observed in powder diagrams for the σ -phase related structures μ , *P*, δ , *R* etc. Thus it appeared that the *M* phase was a new member of the family of σ -phase related structures, of which many members have been found in studies of ternary systems of transition metals with non-transition elements, notably Si (e.g. Bardos & Beck, 1966).

A specimen of the *M* phase of Nb–Ni–Al in stated atomic ratio 48:39:13, annealed at 1140°C for 20 hours, was kindly made available to us by Dr Bill C. Giessen of M.I.T. for single-crystal study. The composition was estimated from ratios of the constituent elements used, with allowance for estimated weight losses; no analysis was made.

Experimental

Approximate values for the cell dimensions were obtained from precession photographs of a single-crystal fragment and were refined by a least-squares fit of the

diffraction peaks observed on a spectrogoniometer trace taken with Cu *K* α radiation, using our program 'LSCELP' (Shoemaker & Shoemaker, 1965). The dimensions of the orthorhombic cell are:*

$$a_0 = 9.303(6), b_0 = 16.266(10), c_0 = 4.933(2) \text{ \AA}.$$

Table 1 is the output of LSCELP and shows the indexing of the powder diagram and the agreement obtained for the observed and calculated θ values. The cell dimensions of the *M* phase are very similar to those for the *P* phase (Shoemaker, Shoemaker & Wilson, 1957): a_0 and c_0 are about 3% larger, but b_0 is about 5% shorter. The systematic absences show that the same glide planes occur in the *M* and the *P* phases but that the types are interchanged between the *a* and *b* axes. The space group for the *P* phase is *Pbnm* (D_{2h}^{16}), and the possible space groups indicated by the extinctions for the *M* phase are *Pnam* (D_{2h}^{16}) and *Pna2*₁ (C_{2v}^9).

The density of the *M* phase as measured with a pycnometer is 8.03(5) g.cm⁻³. The corresponding calculated number of atoms per unit cell is 50.8(0.3). Since the number of atoms is required by these space groups to be a multiple of four, and since our experience with other phases in this family is that the measured density is generally on the low side, the number of atoms per unit cell was taken to be 52.

A trial structure for the *M* phase in space group *Pnam* was obtained which resembles the *P* phase, except that there are fewer atoms per cell (52 for the *M* phase, 56 for the *P* phase) and five-membered rings replace all six-membered rings in the main layers.

The search for a good single crystal in the crushed material was unsuccessful and the intensity data were

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* Figures in parentheses are standard deviations.

collected from a relatively large irregularly shaped crystal fragment (largest dimension 0.1 mm), which gave many spurious reflections from attached randomly oriented crystallites. Intensity data were collected with Mo $K\alpha$ radiation for layer lines $hk0$, $hk1$, $hk2$, and $0kl$, $1kl$, $2kl$ with the precession camera and were visually estimated by comparison with an intensity scale. As in the case of the σ phase and P phase the intensities for layer lines $l \pm 4$ are equal to the corresponding ones for layer lines l (except for normal decline) indicating that the atoms are confined to layers parallel to (001) planes, quartering the cell. Therefore, no attempt was made to collect a complete set of three-dimensional intensity data. No absorption correction was applied, but an extinction correction was introduced at a later stage of the refinement.

Refinement of the structure

The trial structure was first 'refined' without recourse to the intensity data by adjusting the parameters by least squares to fit the distances to values predicted from the coordination numbers of the atoms, the type of the ligands, and the average 12-coordinated radius of the atoms (Shoemaker & Shoemaker, 1964).

The structure factors for this partially refined structure were calculated with the scattering factors tabulated in *International Tables for X-ray Crystallography* (1962). Anomalous dispersion corrections of +0.40 for Ni and -2.18 for Nb were applied (Cromer, 1965). Nb was assumed to occupy the C.N.16, C.N.15 and C.N.14 positions, and Ni, Al and the small amount of remaining Nb were assumed to be uniformly distributed among the C.N.12 positions. The R index for this structure was 0.24.

The refinement was carried out by several cycles of full-matrix least-squares analysis on the IBM 7094 computer with the program ORFLS (Busing, Martin & Levy, 1962). The function minimized was $\sum w(F_o - F_c)^2$. The standard errors in the structure amplitudes, on which the weights were based, were set equal to 10 for all reflections with $|F_o| \leq 100$ and to $0.10|F_o|$ for $|F_o| > 100$. In the first cycles the 420 observed reflections were used and 36 parameters were varied (one scale factor; for each atom x , y and isotropic B ; and for atoms X and XI, also z). Since the calculated struc-

ture factors for the strong reflections were consistently higher than the observed values, a secondary extinction correction was applied of the form $F_o^{corr} = F_o(1 + gF_o^2)$ with $g = 1.33 \times 10^{-6}$. No high intensities occur beyond 2θ of about 52° and g was therefore assumed to be independent of 2θ (Zachariasen, 1963). The final R index ($\sum |AF|/\sum |F_o|$, observed reflections only) was 0.115. Including about 400 non-observed reflections with $F_o = \frac{1}{2}F_{min}$ the R index is 0.178. No structure factors calculated for the non-observed reflections exceed significantly the estimated values of F_{min} . The calculated structure factors are substantially higher than the observed ones for about a dozen strong reflections, principally with $l=2$; it is possible that the scale factor for this layer is somewhat in error.

The final parameters and their standard deviations are listed in Table 2. The z parameters for atoms X and XI are not significantly different from zero (the standard deviations in these parameters are about 0.01 Å). The temperature factors listed in Table 2 were obtained in the previous cycle. Because of the strong interactions between temperature factors and occupancy factors, average values for the temperature factors for atoms of a particular coordination type were taken and held constant in the last cycle, in which the scattering-factor multipliers and the positional parameters but not the overall scale factor were varied. In view of this procedure, the standard deviations given in Table 2 do not fully represent the uncertainties in these parameters. The f_o values listed in Table 2 show that the positions with C.N.14, C.N.15, and C.N.16 are occupied by Nb (f_o for Nb after anomalous dispersion correction is 38.8). The number of atoms in such positions per unit cell (24) is about the same as the number of Nb atoms per unit cell indicated by the overall composition (25). One C.N.15 position (position no. IV with $f_o = 35.5$) may have some Ni and/or some Al admixed in it, but the difference is perhaps not significant. The C.N.12 positions are occupied by essentially the same mixture of Ni and Al (f_o for 0.75 Ni and 0.25 Al is 24.4), except that position VII has a higher scattering factor, indicating either that it is rich in Ni (f_o for Ni is 28.4) or that it contains some Nb. Apparently, no position is rich in Al.

The observed and calculated structure factors (calculated with the parameters given in Table 2, except

Table 2. Atomic parameters for the M phase

Atom	Position	x	y	z	B	f_o	C.N.
I	4(c)	0.0593 (8)	0.1494 (4)	$\frac{1}{4}$	0.63 (0.12)	37.4 (0.9)	14
II	4(c)	0.2996 (8)	0.3984 (4)	$\frac{1}{4}$	0.84 (0.12)	37.7 (0.9)	14
III	4(c)	0.5242 (8)	0.5410 (4)	$\frac{1}{4}$	0.88 (0.13)	38.7 (0.9)	15
IV	4(c)	0.6164 (9)	0.7068 (4)	$\frac{1}{4}$	0.88 (0.13)	35.5 (0.9)	15
V	4(c)	0.0144 (9)	0.4482 (4)	$\frac{1}{4}$	1.19 (0.13)	37.2 (0.9)	16
VI	4(c)	0.8388 (9)	0.2938 (4)	$\frac{1}{4}$	1.14 (0.13)	38.9 (0.9)	16
VII	4(c)	0.0714 (12)	0.6225 (5)	$\frac{1}{4}$	0.72 (0.13)	30.4 (0.8)	12
VIII	4(c)	0.3255 (12)	0.6697 (6)	$\frac{1}{4}$	0.77 (0.16)	25.6 (0.8)	12
IX	4(c)	0.8168 (12)	0.5778 (6)	$\frac{1}{4}$	0.66 (0.16)	26.3 (0.8)	12
X	8(d)	0.1118 (8)	0.2951 (4)	0.0048 (23)	0.63 (0.12)	23.4 (0.6)	12
XI	8(d)	0.2550 (7)	0.5450 (4)	-0.0031 (18)	0.75 (0.12)	26.0 (0.6)	12

that *B*'s were averaged as stated above) are given in Table 3. An analysis of *R* as a function of *l* did not suggest that the agreement could be improved by abandoning the mirror planes and formulating a structure in the lower space group *Pna2*₁. It was therefore concluded that *Pnam* is the correct space group.

Description of the structure

The structure is shown, in projection down the *c* axis, in Fig. 1. Atoms X and XI are located at $z=0$ and $z=\frac{1}{2}$ and all other atoms are situated in the mirror planes at $z=\frac{1}{4}$ and $z=\frac{3}{4}$ perpendicular to the *z* axis. In the σ phase, the atoms in the main layers form hexagons and triangles. All the atoms between the main layers center the hexagons in projection and are therefore 14-coordinated. In the *P* phase the atoms in the main layers form hexagons, pentagons and triangles and half of the atoms between the main layers now center pentagons and are therefore 12-coordinated. In the *M* phase the atoms in the main layers form only pentagons and triangles and all the in-between atoms have now C.N.12.

The coordination polyhedra are of the same type as those found previously in the *P* phase (Shoemaker, Shoemaker & Wilson, 1957), the *R* phase (Komura, Sly & Shoemaker, 1960), the μ phase (Forsyth & d'Alte da Veiga, 1962), and the δ phase (Shoemaker & Shoemaker, 1963).

The coordination numbers are 16, 15, 14 and 12; all coordination shells are triangulated and all the interstices are tetrahedral. No C.N.13 polyhedra such as are found in the χ phase (α -Mn) occur. The symmetries of the coordination polyhedra have been described by Kasper (1956), by Frank & Kasper (1958, 1959) and by Shoemaker, Shoemaker & Wilson (1957). These structures represent close packings of atoms of different sizes.

The percentage of atoms with a particular coordination type is the same for the *M* phase as for the μ phase: it is 15.4% for each of the coordination types C.N.16, C.N.15, and C.N.14 and 53.8% for C.N.12.

An analysis of the structure of the *M* phase in terms of pentagon-strips and triangle-strips (see bottom of Fig. 1) shows that it is included among the structures predicted by Frank & Kasper (1959) in their analysis of complex alloy structures of this kind. This was at first not recognized since this structure is listed as entry 9 of their Table 3, ostensibly describing structures containing two *different* kinds of layer. This entry, and one other (the third), both with numerical symbol (2,2), are, however, special cases and actually contain *equivalent* layers resulting in symmetry higher than the stated *P2*. The number of atoms per unit cell for the predicted *M* phase is correctly given as 52, but the number of atoms with coordinations 14 and 15 is 8 in each case rather than the given 9 and 7.

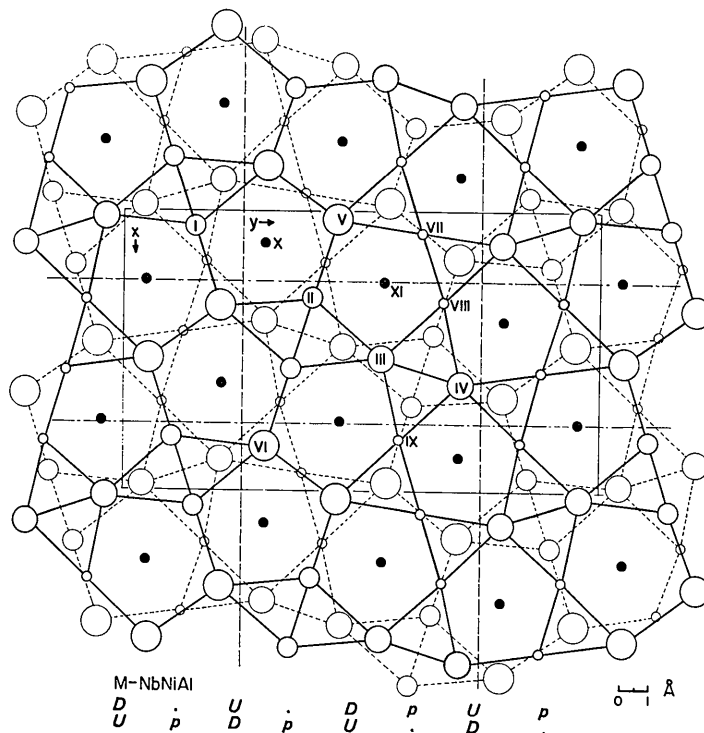


Fig. 1. The crystal structure of the *M* phase, Nb-Ni-Al, in projection on (001). Light circles joined by dashed lines are atoms on one main layer at $z=\frac{3}{4}$, heavy circles joined by solid lines are atoms on the other main layer at $z=\frac{1}{4}$. Each solid black circle represents two atoms in superposition on secondary layers at about $z=0$ and $z=\frac{1}{2}$. Increasing size of circles indicates increase in C.N. from 12 to 16.

In this structure there are pentagon-strips parallel to the *a* axis that have triangle-strips on both sides in the same layer. The pentagon-strips immediately above and below in the adjacent main layers do not have triangle-strips on either side. Therefore these pentagons are larger than the ones in the first layer. Fig. 1 shows that the atoms in the larger pentagons are indeed the ones with the larger coordination numbers.

The *M* phase is closely related to the μ phase, in which the atoms in the main layers normal to $[110]_{\text{hex}}$ also form pentagons and triangles (Fig. 2). In fact, the *M* phase contains blocks of μ -phase structure. This also follows from comparison of the Frank-Kasper letter symbols of these structures given at the bottom of Figs. 1 and 2.

Frank & Kasper (1959) have derived many other known and unknown structures of hexagonal, trigonal, or cubic symmetry from the μ phase by different ways of stacking the hexagonal layers normal to $[001]_{\text{hex}}$. Fig. 2 suggests an alternative way of representing this group of structures. It appears that in these structures the atoms are arranged in layers perpendicular to $[110]$, consisting of only pentagons and triangles, with atoms between the layers forming continuous rows of C.N.12 atoms in the $[110]$ direction. Fig. 3 shows the pentagon layers for Zr_4Al_3 (Wilson, Thomas & Spooner, 1960) and for the Laves phases: MgCu_2 and MgZn_2 . Other ways of stacking the hexagonal layers or joining up the pentagons (Fig. 4) lead to the Laves phase MgNi_2 , the 5- and 9-layer structures described by Komura (1962) and other hypothetical structures.

The secondary layers of the *M* phase consist of a net that is a combination of $4.3.4.3^2$ and $4^2.3^3$ tessellations, topologically the same as in the *P* phase. The triangles are approximately equilateral in the *P* phase, but in the *M* phase they deviate significantly from equilateral as in the μ and Laves phases.

Major networks

The 'major networks' of 6-coordinated bonds are given in Fig. 5. As in the *P* phase, there are two non-interconnected three-dimensional nets, but the additional rows of major ligands connecting C.N.14 atoms in the *c* direction are missing in the *M* phase. In the μ phase there is one three-dimensional network with additional non-interconnected planes of major ligands connecting C.N.15 atoms.

Analysis of the interatomic distances

The interatomic distances in the *M* phase are given in Table 4. They were analysed in the same way as was done before for the other σ -phase related phases. From the interatomic distances 5-coordinated (minor) and 6-coordinated (major) radii were derived for each kind of atom by a least-squares fit. There are 56 non-equivalent distances, and 17 radii were derived from these (one for each C.N.12 atom, and two for each C.N.14,

C.N.15, and C.N.16 atom). The results are given in Table 5. The average value for each particular type of ligand is also given, and compared with the value (in parentheses) that was predicted for this radius from the average 12-coordinated radius of the atoms (Shoemaker & Shoemaker, 1964). In the calculation of the average radius the 12-coordinated radius of Al was taken to be 1.43 as derived from elementary Al. The agreement between the observed and predicted radii is very good for the minor radii; there are somewhat larger differences for the major radii, but none larger than 0.05 Å. As mentioned before, the *R* index for the structure arrived at by a least-squares fit of the distances to sums of these predicted radii was 0.24. The largest shift in atom position from this partially 'refined' structure to the final structure occurred for atom V (C.N.16) and is 0.13 Å. In Table 6 are given interatomic distance values calculated from the average least-square radii in Table 5; these may be compared

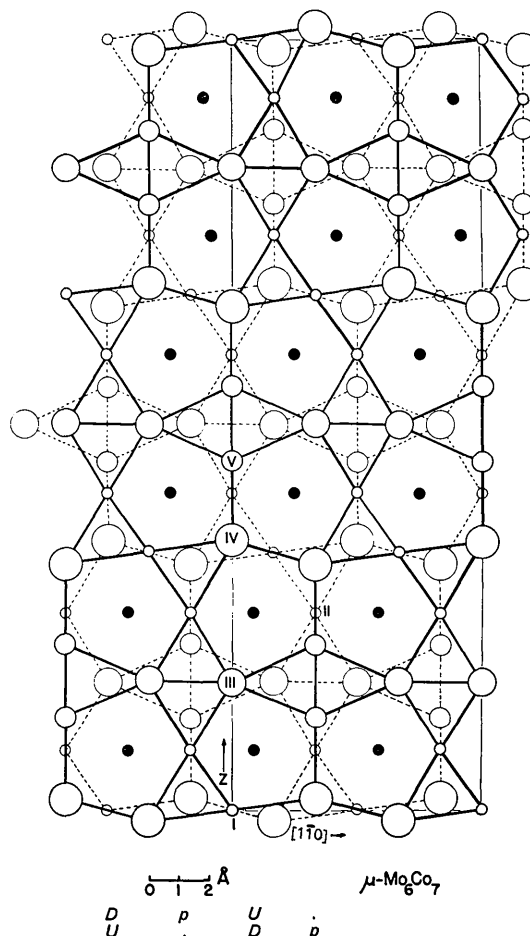


Fig. 2. The crystal structure of the μ phase, Mo_6Co_7 , projected down $[110]$. Solid-line net and broken-line net represent the main layers separated by half of the short diagonal of the hexagonal cell. Each solid black circle represents two atoms in superposition on the secondary layers half way between the main layers. Size of circles as in Fig. 1.

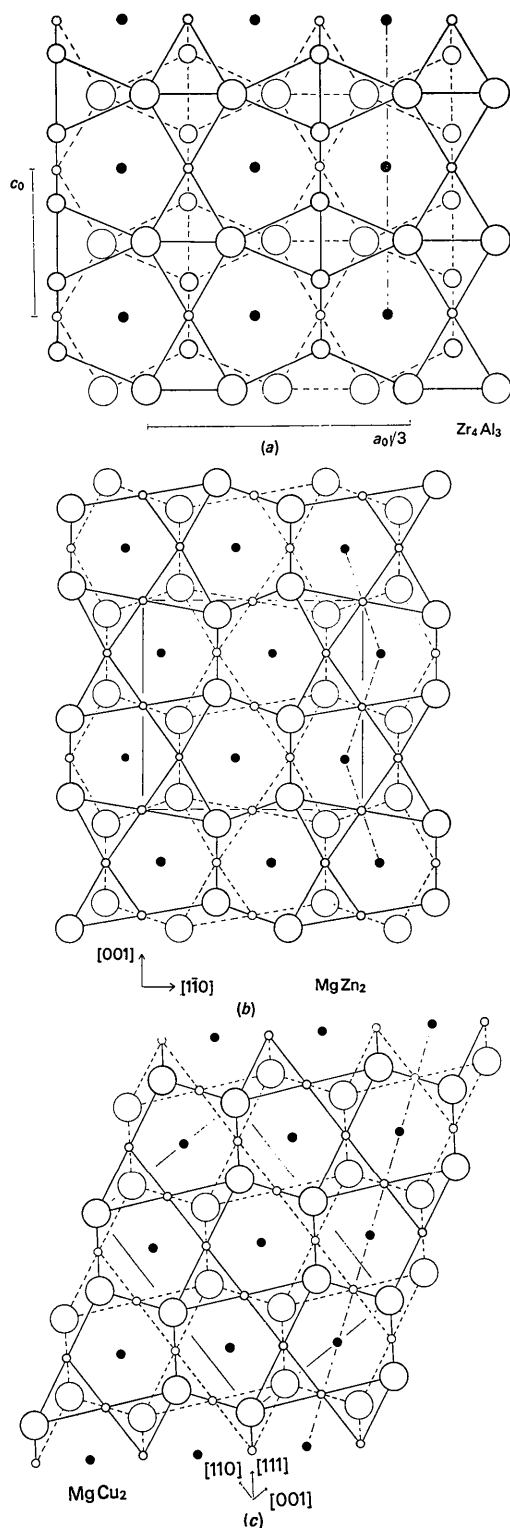


Fig. 3. Pentagon main layers and secondary layers in: (a) Zr_4Al_3 , (b) $MgZn_2$, (c) $MgCu_2$. The main layers all contain the same basic unit consisting of two pentagons sharing one edge parallel to the c axis; these are stacked with triangles. The differences in stacking are indicated by the lines (alternating long and short dashes) joining the interlayer atoms (solid black circles) centering the pentagons in projection.

with the actual distances in Table 4. The largest deviation is 0.122 Å; the mean deviation is 0.035 Å.

The presence of 13 at. % of aluminum causes all distances to be about 4% larger than the corresponding distances in the P phase ($Mo_{42}Cr_{18}Ni_{40}$). In the σ phase Nb_2Al the presence of 33 at. % of aluminum causes all distances to be 10 to 15% larger than those in other σ phases formed of transition elements only (Brown & Forsyth, 1961). Silicon in its ternary transition element phases (Bardos, Bardos & Beck, 1963) displays on the contrary a C.N.12 radius of about 1.20, slightly smaller than the C.N.12 radius for Fe, Co, and Ni.

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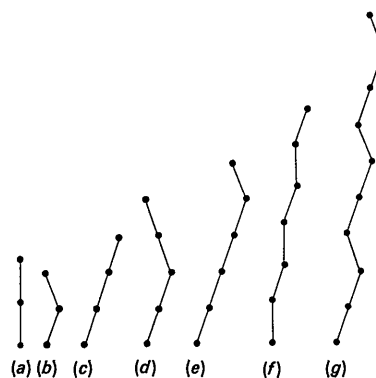


Fig. 4. Lines connecting interlayer atoms (see Fig. 3) in: (a) Zr_4Al_3 , (b) $MgZn_2$, (c) $MgCu_2$, (d) $MgNi_2$, (e) 5-layer Laves phase (Komura), (f) μ phase. (g) 9-layer Laves phase (Komura).

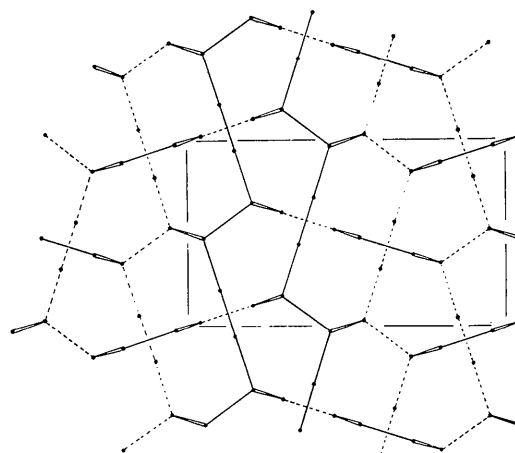


Fig. 5. Frameworks of six-coordinated ligands (major networks) in the M phase. Solid lines at $z = \frac{1}{4}$, broken lines at $z = \frac{3}{4}$. Tapered double lines indicate inclined ligands interconnecting nets on adjacent main layers and forming infinite vertical zigzag rows.

Table 4. Observed interatomic distances for the M phase

Roman numbers refer to atoms
Numbers in brackets represent number of ligands on a given atom having given distance value. For atoms X and XI the number of ligands should be divided by 2 if reading down. (This results from their higher multiplicity.)

	I C.N. 14	II 14	III 15	IV 15	V 16	VI 16	VII 12	VIII 12	IX 12	X 12	XI 12
I		2.538*	3.115	3.103 [2]		3.118		2.710 [2]		2.705 [2]	2.712 [2]
II			3.130 [2]	3.102 [2]	2.775*	3.148			2.722 [2]	2.710 [2]	2.723 [2]
III			3.122								
			3.121 [2]								
III			2.841 [2]*	2.830*				2.793	2.787		2.799 [2]
											2.768 [2]
IV						2.874 [2]*	2.806	2.774	2.808	2.854 [2]	
								2.795		2.824 [2]	
V					2.999 [2]*	2.996*	2.885		2.796	2.914 [2]	3.008 [2]
							2.836 [2]		2.954 [2]		2.789 [2]
VI							2.938 [2]	2.962 [2]		2.813 [2]	3.020 [2]
										2.831 [2]	
VII								2.485	2.478	2.506 [2]	2.463 [2]
VIII										2.465 [2]	2.471 [2]
IX										2.509 [2]	2.433 [2]
X										2.514 [2]	
										2.419 [2]	
XI											2.436 [2]
											2.497 [2]
Largest distance			3.148								
Smallest distance			2.420								
Average distance			2.782								

Standard deviations are estimated to be about 0.015 Å, except for X-X and XI-XI which are estimated to be about 0.020 Å.

* Major ligand.

Table 5. Summary of atomic radii for the M phase

C.N.	Atom	r	r*
12	VII	1.237	
	VIII	1.233	
	IX	1.232	
	X	1.228	
	XI	1.233	
			1.233 (1.24)†
14	I	1.495	1.271
	II	1.501	1.270
		1.498 (1.50)	1.271 (1.22)
15	III	1.589	1.422
	IV	1.592	1.395
		1.591 (1.58)	1.408 (1.37)
16	V	1.657	1.501
	VI	1.673	1.485
		1.665 (1.66)	1.493 (1.52)

† In parentheses: values derived from average C.N.12 radius of the atoms and used in the preliminary refinement of the structure without intensity data.

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Table 6. Interatomic distances in the M phase predicted from average least-squares radii of Table 5

	C.N.12	C.N.14		C.N.15		C.N.16	
C.N.12	2.466						
C.N.14	2.731	2.996	2.541*				
C.N.15	2.824	3.089	2.679*	3.182	2.817*		
C.N.16	2.898	3.163	2.764*	3.256	2.902*	3.330	2.986*